Map Projection and Rectangular Coordinate Systems
Note #4

Map projection
- A systematic transformation of the 3-D spherical surface onto a 2-D plane surface of a flat map.
- Projection formulas are mathematical expressions that convert data from a geographical location (latitude and longitude) on a sphere or ellipsoid to a representative location on a flat surface.
- This projection process inevitably distorts at least one of these properties - shape, area, distance, direction - and often more. So no perfect projection!

Forward map projection
3D spherical coordinates to 2D map Cartesian coordinates.

Inverse map projection
2D map Cartesian coordinates to 3D spherical coordinates.

Reference (nominal) globe
The ellipsoid is reduced to a reference globe from which map projection is generated.

Nominal (principal) scale = nominal globe radius/earth radius.

Map scale = map distance/earth distance

Scale factor (SF): = map scale/nominal scale

Projection Surface
1) Plane (flat) surface -> Azimuthal projection family
- The spherical grid is projected onto a flat planar surface touching the globe. also known as azimuthal projection, zenithal projection or perspective projection.
- In normal orientation, parallels of latitude are concentric circles centered on the pole, and meridians are straight lines that intersect at the pole with their true angles of orientation
- Most often used for mapping circular region, especially polar regions

2) Cylinder -> Cylindrical projection family
- A cylinder is developed by cutting along its length and unrolling it.
- Mercator projection is one of the most common cylindrical projections.
- Suitable for mapping the equatorial area

3) Cone -> Conic projection family
• Features from the reference globe are transferred onto a conic surface, and then developed by making a cut along a meridian before unwrapping it. The meridian opposite to the cut line is known as the central meridian.
• In normal orientation, the projected graticule has straight converging meridians and concentric circular arcs for parallels.
• To ensure the projection accuracy, it is a common practice to cut off the top of the cone, namely, the polar region.
• Suitable for mapping middle latitude areas that have an east-to-west orientation.

Light-Source Position
1) Gnomonic: light source is at the center of the globe.
2) Stereographic: light source is at the point exactly opposite the point of tangency of the projection surface.
3) Orthographic: at a considerable distance (infinite point). Light rays are parallel. Often is used for perspective views of earth, moon, or other planets. The earth appears as if on a photograph from deep space. The resulting map is perspective, but not conformal nor equal-area.

Orientation of Projection Surface
1) Normal (regular) projection
   • The normal orientation for a plane is tangent at the pole (polar azimuthal);
   • A cylinder is normally orientated so that it is tangent along equator (equitorial).
   • A cone is normally oriented so that it is tangent along a parallel, with its apex over the pole, in alignment with the axis of rotation.
2) Transverse projection
   • The projection surface is turned 90 degree from normal.
   • In a transverse projection on a plane, the plane is tangent at the equator (equitorial).
   • In a transverse cylindrical, the cylinder is tangent along a meridian;
   • A transverse conic is not frequently seen.
3) Oblique projection:
   The project surface lies at an angle somewhere between the normal and transverse position.

Tangency of Projection Surface

Tangent projection
• The projection surface is tangent to the globe.
• A planar surface is tangent to globe at one point only.
• Tangential cones and cylinders contact the globe along a line.

Secant projection
• The projection surface intersects the globe instead of merely touching its surface.
• A planar surface intersects the globe, forming a small circle along the intersection line.
A cone or cylinder intersects the globe, resulting in two small circles along the intersection lines.

**Standard point**
- The point at which a planar surface touches the globe.
- Only one standard point exists for planar tangent projection, which is identified by a central longitude and central latitude.
- Directions from the standard point are accurate. Great circles passing through standard point are represented by straight lines, thus the shortest distance between the standard point to any other points on the map is a straight line.
- Distortions of area and angle around the standard point have a circular pattern, and increases with the distance to the standard point.

**Standard line (line of true scale)**
- The line along which projection surface touches or intersects the globe are called standard lines.
- There is one standard line when a planar surface intersects the globe, or a cone or cylinder is tangential to the globe.
- There are two standard lines when a cone or cylinder intersects the globe.
- Along the standard line, the map has no distortion, and the map scale is identical to the nominal globe.
- In general, geometric distortion increases with the distance to the standard lines.

**Desired Geometric Properties for Projection**

**Distortion:**
- A globe is the only correct representation of the geometric properties of the spherical earth’s surface.
- No map projection can maintain all the geometric properties of the nominal globe (distance, area, shape, and direction) simultaneously.

**Major properties**

1) **Equal-area (Equivalence):**
- A unit area drawn anywhere on the map always represents the same number of square units on the globe’s surface. For example, areas of similarly bounded quadrilaterals maintain correct area properties.
- To retain equivalence, scale changes that occur in one direction must be offset by suitable changes of the scale in the opposite direction.
- Linear or distance distortion often occur, and shape is often quite skewed.
- The intersections of meridians and parallels are not at right angles.
- Retaining areal relationships is especially important on maps used to represent the area extent of various phenomena on the earth’s surface.

2) **Conformal (orthomorphic):**
- Conformal imply “correct form or shape”.
• The shapes of small areas are preserved so that map features can be recognized by their distinctive shapes. But the shapes of large mapped regions may be severely distorted.
• Meridians intersect parallels at right angles.
• The scale is the same in all directions at any given point, but the scale may change from point to point.
• Maps to be used for recording motion and angular relationships, such as navigation charts, meteorological charts.

Minor properties
3) Equi-distant
• The length of a straight line between two points on a map represent the correct great-circle distance between the same points on the earth. The path of a great circle is the shortest distance between two points on the earth’s surface.
• Distance can be held true from one to all other points, or from a few points to others, but not from all points to all other points.
• Scale will be uniform along the lines whose distances are true.

4) Azimuthal (direction)
• A straight line drawn between the central point (standard point) to all other points shows the true great-circle route and azimuthal direction from the central point to all other points.
• Directions from points other than the central point (standard) to other points are not accurate.

Visualizing and Determining Distortion Distribution
1) Plot a geometrical figure (square or circle) or familiar object (a person’s head) at several locations on the projection graticule.

2) Tissot’s indicatrix: The distortions are characterized by these qualities:
   - Maximum angular distortion=$2\omega$
   - Scale along the ellipse major axis=$a$
   - Scale along the ellipse minor axis=$b$
   - Maximum areal distortion=$S$

Selection of Map Projections
Factors Influencing the Projection Choice
1) The primary use of the map and desired geometric properties to be preserved
   • Equal-area cylindrical projections are often used to show the world-wide distribution of a variety of geographic phenomena.
   • Presentation maps are usually conformal projections, although compromise and equal-area projections can also be used.
   • Navigational maps are usually Mercator, true direction and/or equidistant.

2) The locations of the area to be mapped
   • Azimuthal projection is often used to map polar region
   • Conic projection is often used to map mid-latitude regions;
• Cylindrical projection is often used to map equatorial region.

3) The Extent of the area to be mapped
• As the mapped area increases to sub-continental, distortion becomes a significant problem.
• As mapped areas become smaller in extent, the selection of the projection becomes less critical. Potential scale errors begin to drop off considerably.

4) Predominant orientation of the area to be mapped
• Albers Equal-area Conic, Lambert Conformal Conic are often used to map the countries that are mainly east-west in extent.
• Transverse Mercator is used to map the states that are mainly north-south in extent.
• Azimuthal projection is better for mapping the area with a circular shape.

Projection Selection
1) World projections
Conformal: MERCATOR, TRANSVERSE, OBLIQUE_MERCATOR
Equal-Area: CYLINDRICAL, ECKERT, MOLLWEIDE, SINUSOIDAL
Equidistant: AZIMUTHAL
Straight Rhumb lines: MERCATOR
Compromise: ROBINSON, MILLER

2) Hemisphere Projections
Conformal: STEREOGRAPHIC, POLAR
Equal-Area: LAMBERT_AZIMUTH
Equidistant: AZIMUTHAL
Global look: ORTHOGRAPHIC

3) Continent or smaller region projections
Predominantly east-west along Equator
Conformal: MERCATOR
Equal-Area: CYLINDRICAL, Mollweide, Sinusoidal

Predominantly east-west at mid-latitude
Conformal: LAMBERT
Equal-Area: ALBERS

Predominantly north-south
Conformal: TRANSVERSE, UTM

Predominantly oblique
Conformal: OBLIQUE_MERCATOR

Equal extent in all directions
Conformal: POLAR, STEREOGRAPHIC, UPS
Equal-Area: LAMBERT_AZIMUTH
Straight great-circle routes: GNOMONIC

Correct scale
Between points: TWO_POINT_EQUIDISTANT
Along meridians: AZIMUTHAL (polar aspect), EQUIDISTANT, EQUIRECTANGULAR, SIMPLE_CONIC
Along parallels: POLYCONIC, SINUSOIDAL, BONNE

Projections Used in the US

Albers Equal-area Conic:
- This is an equal-area conic projection having two standard parallels, but not conformal, perspective, or equidistant.
- Well suited for large countries that mainly east-west in extent.
- USGS uses 29.5N and 45.5N as the standard parallels to map the conterminous US. Maximum scale error is approximately 1.25% over an area the size of the US.
- Meridians are straight lines that intersect parallels at right angle. Parallels are concentric circles, making construction relatively easy.
- This was exclusively used in the National Atlas of the United States of America at scales of 1:7,500,000, 1:17,000,000 and 1:34,000,000. It is also used by the Census Bureau for their base maps.

Lambert Conformal Conic
- Map is conformal, but not perspective, equal-area, or equidistant.
- Distances are true only along standard lines and reasonably accurate elsewhere in limited regions.
- Directions are reasonably accurate, and distortion of shapes and areas minimal at the standard lines.
- One of the most widely used map projections in the US. Used by USGS for many 7.5 and 1.5-minute topographic maps and for the State Base Map series for the 48 conterminous states with standard parallels of 33N and 45N.
- Also used to show other countries or regions that is mainly east-west in extent.

Transverse Mercator
- Transverse cylindrical projection; The map is conformal, the shapes and angles within any small area are essentially true.
- Distortion of distances and size of areas increases outside the 15 degree of the central meridian.
- Graticule spacing increases away from central meridian.
- Used by USGS for many quadrangle maps at scales from 1:24,000 to 1:250,000. Also used for mapping large areas that are mainly north-south in extent.

Mercator
- One of the most common cylindrical projections. This projection is conformal and display true direction along straight line.
- The equator is the tangent line. Meridians and parallels intersect at right angles.
• The meridian is equally spaced, while the spacing between parallels increase towards the poles.
• Solve a major problem of early navigation. Anywhere in any direction a straight line represent a constant bearing. Rhumb lines are straight lines.
• But, on a Mercator chart, Alaska and Brazil appears to be about the same size, but actually Brazil is more than five times larger. So never ever measure areas on Mercator’s projection!

Robinson:
• Pseudocylindric projection
• Uses tabular coordinates rather than mathematical formulas to make the world “look right”.
• Better balance of size and shape of high-latitude lands than Mercator, Mollweide.
• Used in Goode’s atlas, adopted for National Geographic’s world maps.

Common Map Projections Used in Texas
Texas State Mapping System (TSMS)
Projection: Lambert Conformal Conic
Spheroid: Clarke GRS 80
Datum: North American Datum of 1983 (NAD83)
Longitude of Origin: 100 degrees West (-100)
Latitude of Origin: 31 degrees 10 minutes North (31.16_)
Standard Parallel #1: 27 degrees 25 minutes North (27.416_)
Standard Parallel #2: 34 degrees 55 minutes North (34.916_)
False Easting: 1,000,000 meters
False Northing: 1,000,000 meters
Units of Measure: meters

Shackelford
Projection: Lambert Conformal Conic
Spheroid: Clarke 1866
Datum: North American Datum of 1927 (NAD27)
Longitude of Origin: 100 degrees West (-100)
Latitude of Origin: 31 degrees 10 minutes North (31.16_)
Standard Parallel #1: 27 degrees 25 minutes North (27.416_)
Standard Parallel #2: 34 degrees 55 minutes North (34.916_)
False Easting: 3,000,000 feet (914,400 meters)
False Northing: 3,000,000 feet (914,400 meters)
Units of Measure: feet (international)
Texas Centric Mapping System/Lambert Conformal (TCMS/LC)
Projection: Lambert Conformal Conic
Spheroid: Clarke GRS 80
Datum: North American Datum of 1983 (NAD83)
Longitude of Origin: 100 degrees West (-100)
Latitude of Origin: 18 degrees North (18)
Standard Parallel #1: 27 degrees 30 minutes (27.5)
Standard Parallel #2: 35 degrees (35)
False Easting: 1,500,000 meters
False Northing: 5,000,000 meters
Units of Measure: meters

Texas Centric Mapping System/Albers Equal Area (TCMS/AEA)
Projection: Albers Equal Area Conic
Spheroid: Clarke GRS 80
Datum: North American Datum of 1983 (NAD83)
Longitude of Origin: 100 degrees West (-100)
Latitude of Origin: 18 degrees North (18)
Standard Parallel #1: 27 degrees 30 minutes (27.5)
Standard Parallel #2: 35 degrees (35)
False Easting: 1,500,000 meters
False Northing: 6,000,000 meters
Units of Measure: meters

UTM Zones in Texas
Zone 13 (west), Zone 14 (central), Zone 15 (east)

Rectangular (Plane) Map Coordinate Systems

The Needs for Rectangular Coordinates Systems
- **Rectangular Coordinate System** is used to locate positions on a large-scale map. It is evolved from Cartesian coordinates applied for military needs.
- Easy plane geometry and trigonometry could be applied.
- A rectangular grid is superimposed over the projected graticule.
- Accurate conversions at points can be made from the plane coordinate system (UTM and SPCS grid) to the projection grid.
- Plane coordinate systems are only used on large-scale maps.
- Most plane coordinate systems are based on only three projections: the transverse Mercator, the polar stereographic, and Lambert conformal conic. The USGS uses three conformal projections to map the states: the Lambert conformal conic for states with long east-west dimensions, the transverse Mercator for states with long north-south dimensions, and oblique Mercator for portion of Alaska. In each case, over small areas these projections cause little or no areal and distance distortion.
Establishment of Rectangular Coordinates Systems

**Cartesian Coordinate system**
- Origin (initial point), a unit of distance, x-axis, y-axis.
- The Cartesian coordinate system uses two axes: one horizontal (x), representing east-west, and one vertical (y), representing north-south.
- Locations of geographic objects are defined relative to the origin, using the notation (x,y).

**Rectangular Coordinate Systems**
- A map projection is made by transforming the spherical surface to a plane.
- A rectangular plane coordinate grid is placed over the map projection. The origin of the coordinate grid is often placed somewhere near the center of interest on the map. The perpendicular axes of the coordinate grid are usually made to coincide with straight-line meridians and/or parallels, if they exist.
- It is a standard practice for large-scale maps to be on conformal projections. Under such projections, coordinate reference grids maintain the accuracy you need to calculate directions and distances.
- Conventional way to specify a pair of coordinates is, first, to give X value called easting, and then Y value called northing.
- To further simplify calculation, both easting and northing coordinates are often made positive by shifting the origin to the southwest corner. To accomplish this, two arbitrary large values, called false easting and false northing, are given to the Y-axis and X-axis origin.

**Universal Transversal Mercator (UTM) Grid System**

**UTM Coordinate System**
- In UTM grid system, the area of the earth between 84°N and 80°S latitude is divided into north-south columns 6° of longitude wide. These columns are called UTM zones. They are numbered from 1 to 60 eastward, beginning at the 180th meridian.
- For UTM grid system, each of the 60 zones is mapped onto one Transverse Mercator projection. Y-axis is coincides with the central meridian of the 6 longitudinal zone.
- Each column is divided into quadrilaterals of 8° of latitude. The rows quadrilaterals are assigned letters C to X consecutively.
- Each quadrilateral is uniquely identified by a number-letter combination.
- Each 6° by 8° quadrilateral is divided into 100,000-meter zones. Within each 100,000 m square, you can specify an easting and a northing.
- The SF is constant along each north-south coordinate grid line, but it varies in the east-west direction. A secant case of the transverse Mercator projection is constructed for each zone to minimize variations in the SF over the entire projection.
- It is a metric system. The meter is the basic unit for measurement. This reference system is theoretically correct to 1-meter resolution. You can calculate distance and directions between two points in a UTM zone to accuracy of one meter in 2,500 meters.
• Widely adopted by topographic maps, satellite imagery, natural resource databases, etc in the US and other countries.

**UGSU Modification of UTM Coordinate System**
• Only the zone number (1 through 60) and an easting and northing value are used to designate a point.
• Within each UTM zone covering part of the United States, the meridian in the zone’s center is given a false easting value of 500,000 meters.
• The equator is given a northing value of 0 meters for the northern hemisphere coordinates and an arbitrary northing value of 10 million meters for the southern hemisphere.

**Universal Polar Stereographic (UPS) Grid System**
• UPS grid system is used to cover the polar regions (south of 80°S and north of 84°N).
• Each circular polar zone is divided in half by the line representing the 0° and 180° meridians.
• In the north polar zone, the west half is designated grid zone Y, the east half as grid zone Z. In south polar zone, the west half is designated A, and the east half B.
• In the polar areas, the false northings and false eastings of the poles are given 2,000,000 m in both zones.
• The 2,000,000 easting coincides with the 0°-180° meridian line. The 2,000,000 northing coincides with the 90°E-90°W meridian line.
• UPS grid zones are divided into 100,000 meter squares as in UMT grid system.
• A secant stereographic projection is used, and standard parallel is 81° latitude. The SFs vary from parallel to parallel. At the pole, the SF is set at 0.994 (shrinked). In the vicinity of 80° latitude, the SF increases to 1.0016.

**State Plane Coordinate (SPC) System**
• Devised by the US Coast and Geodetic Survey (now US Chart and Geodetic Survey) for each of the 50 states. The purpose is to permanently record original land survey monument locations.
• Based on the transverse Mercator or Lambert conformal conic projection. For SPC zones mapped on the transverse Mercator projection, scale is constant north-south along the meridian and varies east-west along the parallels. When Lambert conformal conic is used, scale varies north and south of the curved standard parallels of the projection and is constant east and west along those parallels.
• To keep the unavoidable scale variation to less than one part in 10,000, a state may have two or more overlapping zones. Each of these zones has its own projection and coordinate grid system.
The units used are feet. Tick marks on USGS large-scale topographic maps show locations of the 10,000-foot grids for the SPC zones.

To specify a location using SPC notation, you need to give the state, zone name, easting (X value), and northing (Y value) in feet.

**Reading assignment:**